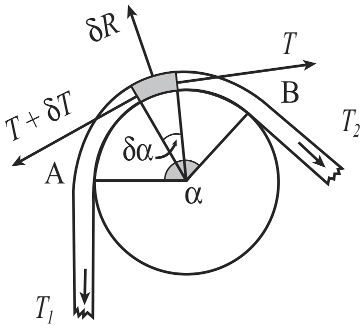
For the mathematically minded

"Sweet e eff a(lpha)?"

By Dr Gina Barney.

When dealing with traction calculations, you may have wondered, as I have, about the origins of the formula below.

$$\frac{T_1}{T_2} \leq e^{f\alpha}$$



It has been said to be due to Euler, and may very well be. I found the explanation, under "rope brake", in a Higher School Certificate textbook I bought 50 years ago !

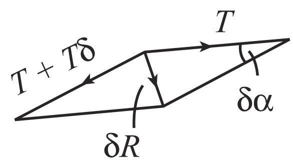
Consider the figure, which shows a rope wrapping a sheave. The car is attached on the LHS and exerts a static force of T_1 at "A". The counterweight is attached on the RHS and exerts a static force of T_2 at "B". At any infinitesimal (i.e very small) element of the rope between "A" and "B", the rope experiences a tension on the RHS of the element of T and a tension on the LHS of the element of $(T + \delta T)$. This tension produces a reaction at right angles of δR.

Slipping occurs when;

$(T + \delta T) - T = f\delta R$ or $\delta R = \delta T/f$ ...(1)

Where f is the coefficient of friction. What is δR?

This can be obtained from the equilibrium of forces diagram.



As $\delta\alpha$ is very, very small, then δR is given by:

$\delta R = T\,\delta\alpha$ ...(2)

Putting (1) and (2) together we get:

$\delta T/f = T\delta\alpha$ or $\delta T/T = f\delta\alpha$

Now adding the contributions of all the elements from $\delta\alpha = 0$ to $\delta\alpha = \alpha$ and δT from T_1 to T_2 we get

$$\int_{T_2}^{T_1} \frac{1}{T}\delta T = f\int_0^{\alpha} \delta\alpha$$

Hence: $log_e \frac{T_1}{T_2} = f\alpha$ which for no slip is $\frac{T_1}{T_2} \leq e^{f\alpha}$ ***QED***

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